THE JOURNAL OF FINANCE • VOL. LI, NO. 1 • MARCH 1996

Multifactor Explanations of Asset Pricing Anomalies

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ABSTRACT

Previous work shows that average returns on common stocks are related to firm characteristics like size, earnings/price, cash flow/price, book-to-market equity, past sales growth, long-term past return, and short-term past return. Because these patterns in average returns apparently are not explained by the CAPM, they are called anomalies. We find that, except for the continuation of short-term returns, the anomalies largely disappear in a three-factor model. Our results are consistent with rational ICAPM or APT asset pricing, but we also consider irrational pricing and data problems as possible explanations.

RESEARCHERS HAVE IDENTIFIED MANY patterns in average stock returns. For example, DeBondt and Thaler (1985) find a reversal in long-term returns; stocks with low long-term past returns tend to have higher future returns. In contrast, Jegadeesh and Titman (1993) find that short-term returns tend to continue; stocks with higher returns in the previous twelve months tend to have higher future returns. Others show that a firm's average stock return is related to its size (ME, stock price times number of shares), book-to-market-equity (BE/ME, the ratio of the book value of common equity to its market value), earnings/price (E/P), cash flow/price (C/P), and past sales growth. (Banz (1981), Basu (1983), Rosenberg, Reid, and Lanstein (1985), and Lakonishok, Shleifer and Vishny (1994).) Because these patterns in average stock returns are not explained by the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), they are typically called anomalies.

This paper argues that many of the CAPM average-return anomalies are related, and they are captured by the three-factor model in Fama and French (FF 1993). The model says that the expected return on a portfolio in excess of the risk-free rate $[E(R_i) - R_f]$ is explained by the sensitivity of its return to three factors: (i) the excess return on a broad market portfolio $(R_M - R_f)$; (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big); and (iii) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks (HML, high minus low). Specifically, the expected excess return on portfolio i is,

$$E(R_i) - R_f = b_i[E(R_M) - R_f] + s_iE(SMB) + h_iE(HML), \tag{1}$$

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where $E(R_M) - R_f$, E(SMB), and E(HML) are expected premiums, and the factor sensitivities or loadings, b_i, s_i, and h_i, are the slopes in the time-series regression,

$$R_i - R_f = \alpha_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + \varepsilon_i. \qquad (2)$$

Fama and French (1995) show that book-to-market equity and slopes on HML proxy for relative distress. Weak firms with persistently low earnings tend to have high BE/ME and positive slopes on HML; strong firms with persistently high earnings have low BE/ME and negative slopes on HML. Using HML to explain returns is thus in line with the evidence of Chan and Chen (1991) that there is covariation in returns related to relative distress that is not captured by the market return and is compensated in average returns. Similarly, using SMB to explain returns is in line with the evidence of Huberman and Kandel (1987) that there is covariation in the returns on small stocks that is not captured by the market return and is compensated in average returns.

The three-factor model in (1) seems to capture much of the cross-sectional variation in average stock returns. FF (1993) show that the model is a good description of returns on portfolios formed on size and BE/ME. FF (1994) use the model to explain industry returns. Here we show that the three-factor model captures the returns to portfolios formed on E/P, C/P, and sales growth. In a nutshell, low E/P, low C/P, and high sales growth are typical of strong firms that have negative slopes on HML. Since the average HML return is strongly positive (about 6 percent per year), these negative loadings, which are similar to the HML slopes for low-BE/ME stocks, imply lower expected returns in (1). Conversely, like high-BE/ME stocks, stocks with high E/P, high C/P, or low sales growth tend to load positively on HML (they are relatively distressed), and they have higher average returns. The three-factor model also captures the reversal of long-term returns documented by DeBondt and Thaler (1985). Stocks with low long-term past returns (losers) tend to have positive SMB and HML slopes (they are smaller and relatively distressed) and higher future average returns. Conversely, long-term winners tend to be strong stocks that have negative slopes on HML and low future returns.

Equation (1), however, cannot explain the continuation of short-term returns documented by Jegadeesh and Titman (1993). Like long-term losers, stocks that have low short-term past returns tend to load positively on HML; like long-term winners, short-term past winners load negatively on HML. As it does for long-term returns, this pattern in the HML slopes predicts reversal rather than continuation for future returns. The continuation of short-term returns is thus left unexplained by our model.

At a minimum, the available evidence suggests that the three-factor model in (1) and (2), with intercepts in (2) equal to 0.0, is a parsimonious description of returns and average returns. The model captures much of the variation in the cross-section of average stock returns, and it absorbs most of the anomalies that have plagued the CAPM. More aggressively, we argue in FF (1993, 1994,

1995) that the empirical successes of (1) suggest that it is an equilibrium pricing model, a three-factor version of Merton's (1973) intertemporal CAPM (ICAPM) or Ross's (1976) arbitrage pricing theory (APT). In this view, SMB and HML mimic combinations of two underlying risk factors or state variables of special hedging concern to investors.

Our aggressive interpretation of tests of (1) has produced reasonable skepticism, much of it centered on the premium for distress (the average HML return). Kothari, Shanken, and Sloan (1995) argue that a substantial part of the premium is due to survivor bias; the data source for book equity (COMPUSTAT) contains a disproportionate number of high-BE/ME firms that survive distress, so the average return for high-BE/ME firms is overstated. Another view is that the distress premium is just data snooping; researchers tend to search for and fixate on variables that are related to average return, but only in the sample used to identify them (Black (1993), MacKinlay (1995)). A third view is that the distress premium is real but irrational, the result of investor over-reaction that leads to underpricing of distressed stocks and overpricing of growth stocks (Lakonishok, Shleifer, and Vishny (1994), Haugen (1995)).

Section VI discusses the competing stories for the successes of the three-factor model. First, however, Sections I to V present the evidence that the model captures most of the average-return anomalies of the CAPM.

I. Tests on the 25 FF Size-BE/ME Portfolios

To set the stage, Table I shows the average excess returns on the 25 Fama-French (1993) size-BE/ME portfolios of value-weighted NYSE, AMEX, and NASD stocks. The table shows that small stocks tend to have higher returns than big stocks and high-book-to-market stocks have higher returns than low-BE/ME stocks.

Table I also reports estimates of the three-factor time-series regression (2). If the three-factor model (1) describes expected returns, the regression intercepts should be close to 0.0. The estimated intercepts say that the model leaves a large negative unexplained return for the portfolio of stocks in the smallest size and lowest BE/ME quintiles, and a large positive unexplained return for the portfolio of stocks in the largest size and lowest BE/ME quintiles. Otherwise the intercepts are close to 0.0.

The F-test of Gibbons, Ross, and Shanken (GRS 1989) rejects the hypothesis that (1) explains the average returns on the 25 size-BE/ME portfolios at the 0.004 level. This rejection of the three-factor model is testimony to the explanatory power of the regressions. The average of the 25 regression R^2 is 0.93, so small intercepts are distinguishable from zero. The model does capture most of the variation in the average returns on the portfolios, as witnessed by the small average absolute intercept, 0.093 percent (about nine basis points) per month. We show next that the model does an even better job on most of the other sets of portfolios we consider.

A comment on methodology is necessary. In the time-series regression (2), variation through time in the expected premiums $E(R_M) - R_f$, E(SMB), and E(HML) in (1) is embedded in the explanatory returns, $R_M - R_f$, SMB, and HML. Thus the regression intercepts are net of (they are conditional on) variation in the expected premiums. We also judge that forming portfolios

Table I

Summary Statistics and Three-Factor Regressions for Simple Monthly Percent Excess Returns on 25 Portfolios Formed on Size and BE/ME: 7/63–12/93, 366 Months

R_f is the one-month Treasury bill rate observed at the beginning of the month (from CRSP). The explanatory returns R_M, SMB, and HML are formed as follows. At the end of June of each year t (1963–1993), NYSE, AMEX, and Nasdaq stocks are allocated to two groups (small or big, S or B) based on whether their June market equity (ME, stock price times shares outstanding) is below or above the median ME for NYSE stocks. NYSE, AMEX, and Nasdaq stocks are allocated in an independent sort to three book-to-market equity (BE/ME) groups (low, medium, or high; L, M, or H) based on the breakpoints for the bottom 30 percent, middle 40 percent, and top 30 percent of the values of BE/ME for NYSE stocks. Six size-BE/ME portfolios (S/L, S/M, S/H, B/L, B/M, B/H) are defined as the intersections of the two ME and the three BE/ME groups. Value-weight monthly returns on the portfolios are calculated from July to the following June. SMB is the difference, each month, between the average of the returns on the three small-stock portfolios (S/L, S/M, and S/H) and the average of the returns on the three big-stock portfolios (B/L, B/M, and B/H). HML is the difference between the average of the returns on the two high-BE/ME portfolios (S/H and B/H) and the average of the returns on the two low-BE/ME portfolios (S/L and B/L). The 25 size-BE/ME portfolios are formed like the six size-BE/ME portfolios used to construct SMB and HML, except that quintile breakpoints for ME and BE/ME for NYSE stocks are used to allocate NYSE, AMEX, and Nasdaq stocks to the portfolios.

BE is the COMPUSTAT book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use redemption, liquidation, or par value (in that order) to estimate the book value of preferred stock. The BE/ME ratio used to form portfolios in June of year t is then book common equity for the fiscal year ending in calendar year $t - 1$, divided by market equity at the end of December of $t - 1$. We do not use negative BE firms, which are rare prior to 1980, when calculating the breakpoints for BE/ME or when forming the size-BE/ME portfolios. Also, only firms with ordinary common equity (as classified by CRSP) are included in the tests. This means that ADR's, REIT's, and units of beneficial interest are excluded.

The market return R_M is the value-weight return on all stocks in the size-BE/ME portfolios, plus the negative BE stocks excluded from the portfolios.

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Summary Statistics									
	Means					Standard Deviations				
Small	0.31	0.70	0.82	0.95	1.08	7.67	6.74	6.14	5.85	6.14
2	0.48	0.71	0.91	0.93	1.09	7.13	6.25	5.71	5.23	5.94
3	0.44	0.68	0.75	0.86	1.05	6.52	5.53	5.11	4.79	5.48
4	0.51	0.39	0.64	0.80	1.04	5.86	5.28	4.97	4.81	5.67
Big	0.37	0.39	0.36	0.58	0.71	4.84	4.61	4.28	4.18	4.89

Table I—*Continued*

Size	Book-to-Market Equity (BE/ME) Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Panel B: Regressions: $R_i - R_f = a_i + b_i(R_M - R_f) + s_iSMB + h_iHML + e_i$									
	a					t(a)				
Small	−0.45	−0.16	−0.05	0.04	0.02	−4.19	−2.04	−0.82	0.69	0.29
2	−0.07	−0.04	0.09	0.07	0.03	−0.80	−0.59	1.33	1.13	0.51
3	−0.08	0.04	−0.00	0.06	0.07	−1.07	0.47	−0.06	0.88	0.89
4	0.14	−0.19	−0.06	0.02	0.06	1.74	−2.43	−0.73	0.27	0.59
Big	0.20	−0.04	−0.10	−0.08	−0.14	3.14	−0.52	−1.23	−1.07	−1.17
	b					t(b)				
Small	1.03	1.01	0.94	0.89	0.94	39.10	50.89	59.93	58.47	57.71
2	1.10	1.04	0.99	0.97	1.08	52.94	61.14	58.17	62.97	65.58
3	1.10	1.02	0.98	0.97	1.07	57.08	55.49	53.11	55.96	52.37
4	1.07	1.07	1.05	1.03	1.18	54.77	54.48	51.79	45.76	46.27
Big	0.96	1.02	0.98	0.99	1.07	60.25	57.77	47.03	53.25	37.18
	s					t(s)				
Small	1.47	1.27	1.18	1.17	1.23	39.01	44.48	52.26	53.82	52.65
2	1.01	0.97	0.88	0.73	0.90	34.10	39.94	36.19	32.92	38.17
3	0.75	0.63	0.59	0.47	0.64	27.09	24.13	22.37	18.97	22.01
4	0.36	0.30	0.29	0.22	0.41	12.87	10.64	10.17	6.82	11.26
Big	−0.16	−0.13	−0.25	−0.16	−0.03	−6.97	−5.12	−8.45	−6.21	−0.77
	h					t(h)				
Small	−0.27	0.10	0.25	0.37	0.63	−6.28	3.03	9.74	15.16	23.62
2	−0.49	0.00	0.26	0.46	0.69	−14.66	0.34	9.21	18.14	25.59
3	−0.39	0.03	0.32	0.49	0.68	−12.56	0.89	10.73	17.45	20.43
4	−0.44	0.03	0.31	0.54	0.72	−13.98	0.97	9.45	14.70	17.34
Big	−0.47	0.00	0.20	0.56	0.82	−18.23	0.18	6.04	18.71	17.57
	R^2					s(e)				
Small	0.93	0.95	0.96	0.96	0.96	1.97	1.49	1.18	1.13	1.22
2	0.95	0.96	0.95	0.95	0.96	1.55	1.27	1.28	1.16	1.23
3	0.95	0.94	0.93	0.93	0.92	1.44	1.37	1.38	1.30	1.52
4	0.94	0.92	0.91	0.88	0.89	1.46	1.47	1.51	1.69	1.91
Big	0.94	0.92	0.87	0.89	0.81	1.19	1.32	1.55	1.39	2.15

periodically on size, BE/ME, E/P, C/P, sales growth, and past returns results in loadings on the three factors that are roughly constant. Variation through time in the slopes is, however, important in other applications. For example, FF (1994) show that because industries wander between growth and distress, it is critical to allow for variation in SMB and HML slopes when applying (1) and (2) to industries.

II. LSV Deciles

Lakonishok, Shleifer, and Vishny (LSV 1994) examine the returns on sets of deciles formed from sorts on BE/ME, E/P, C/P, and five-year sales rank. Table II summarizes the excess returns on our versions of these portfolios. The portfolios are formed each year as in LSV using COMPUSTAT accounting data for the fiscal year ending in the current calendar year (see table footnote). We then calculate returns beginning in July of the following year. (LSV start their returns in April.) To reduce the influence of small stocks in these (equal-weight) portfolios, we use only NYSE stocks. (LSV use NYSE and AMEX.) To be included in the tests for a given year, a stock must have data on all the LSV variables. Thus, firms must have COMPUSTAT data on sales for six years before they are included in the return tests. As in LSV, this reduces biases that might arise because COMPUSTAT includes historical data when it adds firms (Banz and Breen (1986), Kothari, Shanken, and Sloan (1995)).

Our sorts of NYSE stocks in Table II produce strong positive relations between average return and BE/ME, E/P, or C/P, much like those reported by LSV for NYSE and AMEX firms. Like LSV, we find that past sales growth is negatively related to future return. The estimates of the three-factor regression (2) in Table III show, however, that the three-factor model (1) captures these patterns in average returns. The regression intercepts are consistently small. Despite the strong explanatory power of the regressions (most R^2 values are greater than 0.92), the GRS tests never come close to rejecting the hypothesis that the three-factor model describes average returns. In terms of both the magnitudes of the intercepts and the GRS tests, the three-factor model does a better job on the LSV deciles than it does on the 25 FF size-BE/ME portfolios. (Compare Tables I and III.)

For perspective on why the three-factor model works so well on the LSV portfolios, Table III shows the regression slopes for the C/P deciles. Higher-C/P portfolios produce larger slopes on SMB and especially HML. This pattern in the slopes is also observed for the BE/ME and E/P deciles (not shown). It seems that dividing an accounting variable by stock price produces a characterization of stocks that is related to their loadings on HML. Given the evidence in FF (1995) that loadings on HML proxy for relative distress, we can infer that low BE/ME, E/P, and C/P are typical of strong stocks, while high BE/ME, E/P, and C/P are typical of stocks that are relatively distressed. The patterns in the loadings of the BE/ME, E/P, and C/P deciles on HML, and the high average value of HML (0.46 percent per month, 6.33 percent per year) largely explain how the three-factor regressions transform the strong positive relations between average return and these ratios (Table II) into intercepts that are close to 0.0.

Among the sorts in Table III, the three-factor model has the hardest time with the returns on the sales-rank portfolios. Recall that high sales-rank firms

Table II

Summary Statistics for Simple Monthly Excess Returns (in Percent) on the LSV Equal-Weight Deciles: 7/63–12/93, 366 Months

At the end of June of each year t (1963–1993), the NYSE stocks on COMPUSTAT are allocated to ten portfolios, based on the decile breakpoints for BE/ME (book-to-market equity), E/P (earnings/price), C/P (cashflow/price), and past five-year sales rank (5-Yr SR). Equal-weight returns on the portfolios are calculated from July to the following June, resulting in a time series of 366 monthly returns for July 1963 to December 1993. To be included in the tests for a given year, a stock must have data on all of the portfolio-formation variables of this table. Thus, the sample of firms is the same for all variables.

For portfolios formed in June of year t, the denominator of BE/ME, E/P, and C/P is market equity (ME, stock price times shares outstanding) for the end of December of year $t-1$, and BE, E, and C are for the fiscal year ending in calendar year $t-1$. Book equity BE is defined in Table I. E is earnings before extraordinary items but after interest, depreciation, taxes, and preferred dividends. Cash flow, C, is E plus depreciation.

The five-year sales rank for June of year t, 5-Yr SR(t), is the weighted average of the annual sales growth ranks for the prior five years, that is,

$$\text{5-Yr SR}(t) = \sum_{j=1}^{5} (6-j) \times \text{Rank}(t-j)$$

The sales growth for year $t-j$ is the percentage change in sales from $t-j-1$ to $t-j$, $\ln[\text{Sales}(t-j)/\text{Sales}(t-j-1)]$. Only firms with data for all five prior years are used to determine the annual sales growth ranks for years $t-5$ to $t-1$.

For each portfolio, the table shows the mean monthly return in excess of the one-month Treasury bill rate (Mean), the standard deviation of the monthly excess returns (Std. Dev.), and the ratio of the mean excess return to its standard error [t(mean) = Mean/(Std. Dev./$365^{1/2}$)]. Ave ME is the average size (ME, in $millions) of the firms in a portfolio, averaged across the 366 sample months.

	Deciles									
	1	2	3	4	5	6	7	8	9	10
BE/ME	**Low**									**High**
Mean	0.42	0.50	0.53	0.58	0.65	0.72	0.81	0.84	1.03	1.22
Std. Dev.	5.81	5.56	5.57	5.52	5.23	5.03	4.96	5.06	5.52	6.82
t(Mean)	1.39	1.72	1.82	2.02	2.38	2.74	3.10	3.17	3.55	3.43
Ave. ME	2256	1390	1125	1037	1001	864	838	730	572	362
E/P	**Low**									**High**
Mean	0.55	0.45	0.54	0.63	0.67	0.77	0.82	0.90	0.99	1.03
Std. Dev.	6.09	5.62	5.51	5.35	5.14	5.18	4.94	4.88	5.05	5.87
t(Mean)	1.72	1.52	1.89	2.24	2.49	2.84	3.16	3.51	3.74	3.37
Ave. ME	1294	1367	1211	1209	1411	1029	1022	909	862	661
C/P	**Low**									**High**
Mean	0.43	0.45	0.60	0.67	0.70	0.76	0.77	0.86	0.97	1.16
Std. Dev.	5.80	5.67	5.57	5.39	5.39	5.19	5.00	4.88	4.96	6.36
t (Mean)	1.41	1.52	2.06	2.37	2.47	2.78	2.93	3.36	3.75	3.47
Ave. ME	1491	1266	1112	1198	990	994	974	951	990	652
5-Yr SR	**High**									**Low**
Mean	0.47	0.63	0.70	0.68	0.67	0.74	0.70	0.78	0.89	1.03
Std. Dev.	6.39	5.66	5.46	5.15	5.22	5.10	5.00	5.10	5.25	6.13
t (Mean)	1.42	2.14	2.45	2.52	2.46	2.78	2.68	2.91	3.23	3.21
Ave. ME	937	1233	1075	1182	1265	1186	1075	884	744	434

Table III

Three-Factor Time-Series Regressions for Monthly Excess Returns (in Percent) on the LSV Equal-Weight Deciles: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the BE/ME, E/P, C/P, and five-year-sales-rank (5-Yr SR) deciles is described in Table II. The explanatory returns, $R_M - R_f$, SMB, and HML are described in Table I. $t(\ )$ is a regression coefficient divided by its standard error. The regression R^2s are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of ten portfolios are all 0.0. $p(GRS)$ is the p-value of GRS, that is, the probability of a GRS value as large or larger than the observed value if the zero-intercepts hypothesis is true.

	Deciles											
	1	2	3	4	5	6	7	8	9	10	GRS	$p(GRS)$
BE/ME	**Low**									**High**		
a	0.08	−0.02	−0.09	−0.11	−0.08	−0.03	0.01	−0.04	0.03	−0.00		
$t(a)$	1.19	−0.26	−1.25	−1.39	−1.16	−0.40	0.15	−0.61	0.43	−0.02	0.57	0.841
R^2	0.95	0.95	0.94	0.93	0.94	0.94	0.94	0.94	0.95	0.89		
E/P	**Low**									**High**		
a	−0.00	−0.07	−0.07	−0.04	−0.03	0.02	0.06	0.09	0.12	0.00		
$t(a)$	−0.07	−1.07	−0.94	−0.52	−0.43	0.24	1.01	1.46	1.49	0.05	0.84	0.592
R^2	0.91	0.95	0.94	0.94	0.94	0.94	0.94	0.94	0.92	0.92		
C/P	**Low**									**High**		
a	0.02	−0.08	−0.07	−0.00	−0.04	0.00	0.00	0.05	0.06	0.01		
b	1.04	1.06	1.08	1.06	1.05	1.04	0.99	1.00	0.98	1.14		
s	0.45	0.50	0.54	0.51	0.55	0.50	0.53	0.48	0.57	0.92		
h	−0.39	−0.18	0.07	0.11	0.23	0.31	0.36	0.50	0.67	0.79		
$t(a)$	0.22	−1.14	−1.00	−0.04	−0.51	0.00	0.06	0.72	0.92	0.14	0.49	0.898
$t(b)$	51.45	61.16	62.49	64.15	59.04	61.28	60.02	63.36	58.92	46.49		
$t(s)$	15.56	20.32	22.11	21.57	21.49	20.72	22.19	21.17	24.13	26.18		
$t(h)$	−12.03	−6.52	2.56	4.28	7.85	11.40	13.52	19.46	24.88	19.74		
R^2	0.93	0.95	0.95	0.95	0.94	0.94	0.94	0.94	0.94	0.92		
5-Yr SR	**High**									**Low**		
a	−0.21	−0.06	−0.03	−0.01	−0.04	−0.02	−0.04	0.00	0.04	0.07		
b	1.16	1.10	1.09	1.03	1.03	1.03	1.00	0.99	0.99	1.02		
s	0.72	0.56	0.52	0.49	0.52	0.51	0.50	0.57	0.67	0.95		
h	−0.09	0.09	0.21	0.20	0.24	0.33	0.33	0.36	0.47	0.50		
$t(a)$	−2.60	−0.97	−0.49	−0.20	−0.61	−0.25	−0.66	0.07	0.47	0.60	0.87	0.563
$t(b)$	59.01	70.59	67.65	65.34	56.68	68.89	62.49	54.12	50.08	34.54		
$t(s)$	25.69	25.11	22.59	21.65	20.15	23.64	21.89	21.65	23.65	22.34		
$t(h)$	−2.88	3.55	8.05	7.98	8.07	13.63	12.80	12.13	14.78	10.32		
R^2	0.95	0.96	0.95	0.95	0.93	0.95	0.94	0.93	0.92	0.87		

(strong past performers) have low future returns, and low sales-rank firms (weak past performers) have high future returns (Table II). The three-factor model of (1) captures most of this pattern in average returns, largely because low sales-rank stocks behave like distressed stocks (they have stronger loadings on HML). But a hint of the pattern is left in the regression intercepts. Except for the highest sales-rank decile, however, the intercepts are close to 0.0. Moreover, although the intercepts for the sales-rank deciles produce the largest GRS F-statistic (0.87), it is close to the median of its distribution when the true intercepts are all 0.0 (its p-value is 0.563). This evidence that the three-factor model describes the returns on the sales-rank deciles is important since sales rank is the only portfolio-formation variable (here and in LSV) that is not a transformed version of stock price. (See also the industry tests in FF (1994).)

III. LSV Double-Sort Portfolios

LSV argue that sorting stocks on two accounting variables more accurately distinguishes between strong and distressed stocks, and produces larger spreads in average returns. Because accounting ratios with stock price in the denominator tend to be correlated, LSV suggest combining sorts on sales rank with sorts on BE/ME, E/P, or C/P. We follow their procedure and separately sort firms each year into three groups (low 30 percent, medium 40 percent, and high 30 percent) on each variable. We then form sets of nine portfolios as the intersections of the sales-rank sort and the sorts on BE/ME, E/P, or C/P. Confirming their results, Table IV shows that the sales-rank sort increases the spread of average returns provided by the sorts on BE/ME, E/P, or C/P. In fact, the two double-whammy portfolios, combining low BE/ME, E/P, or C/P with high sales growth (portfolio 1-1), and high BE/ME, E/P, or C/P with low sales growth (portfolio 3-3), always have the lowest and highest post-formation average returns.

Table V shows that the three-factor model has little trouble describing the returns on the LSV double-sort portfolios. Strong negative loadings on HML (which has a high average premium) bring the low returns on the 1-1 portfolios comfortably within the predictions of the three-factor model; the most extreme intercept for the 1-1 portfolios is -6 basis points (-0.06 percent) per month and less than one standard error from 0.0. Conversely, because the 3-3 portfolios have strong positive loadings on SMB and HML (they behave like smaller distressed stocks), their high average returns are also predicted by the three-factor model. The intercepts for these portfolios are positive, but again quite close to (less than 8 basis points and 0.7 standard errors from) 0.0.

The GRS tests in Table V support the inference that the intercepts in the three-factor regression (2) are 0.0; the smallest p-value is 0.284. Thus, whether the spreads in average returns on the LSV double-sort portfolios are caused by risk or over-reaction, the three-factor model in equation (1) describes them parsimoniously.

IV. Portfolios Formed on Past Returns

DeBondt and Thaler (1985) find that when portfolios are formed on long-term (three- to five-year) past returns, losers (low past returns) have high

Table IV

Summary Statistics for Excess Returns (in Percent) on the LSV Equal-Weight Double-Sort Portfolios: 7/63–12/93, 366 Months

At the end of June of each year t (1963–1993), the NYSE stocks on COMPUSTAT are allocated to three equal groups (low, medium, and high: 1, 2, and 3) based on their sorted BE/ME, E/P, or C/P ratios for year $t - 1$. The NYSE stocks on COMPUSTAT are also allocated to three equal groups (high, medium, and low: 1, 2, and 3) based on their five-year sales rank. The intersections of the sales-rank sort with the BE/ME, E/P, or E/P sorts are then used to create three sets of nine portfolios (BE/ME & Sales Rank, E/P & Sales Rank, C/P & Sales Rank). Equal-weight returns on the portfolios are calculated from July to the following June. To be included in the tests for a given year, a stock must have data on all of the portfolio-formation variables. The sample of firms is thus the same for all variables. BE/ME (book-to-market equity), E/P (earnings/price), C/P (cashflow/price), and five-year sales rank are defined in Table II. The 1-1 portfolios contain strong firms (high sales growth and low BE/ME, E/P, or C/P), while the 3-3 portfolios contain weak firms (low sales growth and high BE/ME, E/P, or C/P).

For each portfolio, the table shows the mean monthly return in excess of the one-month Treasury bill rate (Mean), the standard deviation of the monthly excess returns (Std. Dev.), and the ratio of the mean excess return to its standard error [t(mean) = Mean/(Std. Dev./$365^{1/2}$)]. Ave. ME is the average size (ME, in \$millions) of the firms in a portfolio, averaged across the 366 sample months. Count is the average across months of the number of firms in a portfolio.

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3
BE/ME and Sales Rank									
Mean	0.47	0.49	0.52	0.64	0.69	0.74	0.93	0.94	1.11
Std. Dev.	5.95	5.19	5.63	5.75	4.97	5.02	6.45	5.59	5.99
t(Mean)	1.52	1.81	1.77	2.11	2.66	2.83	2.76	3.20	3.55
Count	151	109	41	106	180	116	49	118	146
Ave. ME	1530	1867	1061	723	1110	866	482	655	445
E/P and Sales Rank									
Mean	0.41	0.47	0.77	0.63	0.72	0.82	0.80	0.86	1.06
Std. Dev.	6.02	5.44	5.76	5.76	4.94	4.96	6.08	5.33	5.90
t(Mean)	1.31	1.66	2.57	2.10	2.80	3.16	2.51	3.08	3.43
Count	114	98	68	105	163	104	87	145	131
Ave. ME	1394	1524	739	1103	1355	928	651	754	506
C/P and Sales Rank									
Mean	0.44	0.45	0.70	0.62	0.71	0.83	0.85	0.91	1.06
Std. Dev.	6.03	5.26	5.76	5.80	5.01	5.09	6.13	5.34	5.90
t(Mean)	1.40	1.64	2.33	2.03	2.70	3.10	2.64	3.27	3.44
Count	122	107	62	106	166	115	78	134	125
Ave. ME	1365	1527	648	1067	1187	796	615	881	616

future returns and winners (high past returns) have low future returns. In contrast, Jegadeesh and Titman (1993) and Asness (1994) find that when portfolios are formed on short-term (up to a year of) past returns, past losers tend to be future losers and past winners are future winners.

Table VI shows average returns on sets of ten equal-weight portfolios formed monthly on short-term (11 months) and long-term (up to five years of) past returns. The results for July 1963 to December 1993 confirm the strong continuation of short-term returns. The average excess return for the month

Table V

Three-Factor Regressions for Monthly Excess Returns (in Percent) on the LSV Equal-Weight Double-Sort Portfolios: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the double-sort portfolios is described in Table IV. BE/ME (book-to-market equity), E/P (earnings/price), C/P (cashflow/price), and five-year sales rank are described in Table II. The 1-1 portfolios contain strong firms (high sales growth and low BE/ME, E/P, or C/P), while the 3-3 portfolios contain weak firms (low sales growth and high BE/ME, E/P, or C/P). $t()$ is a regression coefficient divided by its standard error. The regression R^2 are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the nine regression intercepts for a set of double-sort portfolios are all 0.0. p(GRS) is the p-value of GRS.

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3	GRS	p (GRS)
BE/ME & Sales Rank											
a	−0.00	0.00	−0.06	−0.19	−0.00	0.00	−0.19	−0.07	0.07		
b	1.10	1.03	1.00	1.12	1.00	0.99	1.17	1.06	1.01		
s	0.49	0.31	0.55	0.63	0.48	0.50	0.87	0.74	0.97		
h	−0.33	−0.14	−0.04	0.31	0.25	0.32	0.75	0.70	0.68		
$t(a)$	−0.10	0.12	−0.57	−2.59	−0.07	0.12	−1.64	−0.94	0.69	1.22	0.284
$t(b)$	71.67	67.85	35.65	61.81	67.36	51.00	41.29	54.45	38.46		
$t(s)$	22.30	14.32	13.77	24.42	22.44	18.18	21.36	26.62	25.76		
$t(h)$	−13.19	−5.74	−0.94	10.57	10.33	10.17	16.30	22.31	15.91		
R^2	0.96	0.95	0.86	0.94	0.95	0.92	0.89	0.93	0.89		
E/P & Sales Rank											
a	−0.06	−0.06	0.02	−0.09	0.03	0.06	−0.19	−0.06	0.06		
b	1.11	1.04	1.02	1.11	1.01	0.99	1.13	1.04	1.00		
s	0.48	0.45	0.74	0.58	0.43	0.48	0.82	0.65	0.92		
h	−0.34	−0.12	0.18	0.14	0.25	0.39	0.53	0.58	0.61		
$t(a)$	−0.89	−0.87	0.24	−1.23	0.53	0.81	−2.10	−0.82	0.59	1.06	0.394
$t(b)$	62.12	56.09	41.52	58.97	67.48	53.80	51.32	59.05	37.61		
$t(s)$	18.61	17.04	21.07	21.30	20.18	18.13	26.08	25.66	23.98		
$t(h)$	−11.56	−3.86	4.41	4.50	10.46	12.88	14.92	20.49	14.19		
R^2	0.95	0.94	0.90	0.94	0.95	0.92	0.93	0.94	0.89		
C/P & Sales Rank											
a	−0.02	−0.06	−0.02	−0.14	0.00	0.07	−0.17	−0.02	0.04		
b	1.11	1.01	1.02	1.12	1.02	1.00	1.13	1.04	1.00		
s	0.46	0.42	0.72	0.63	0.46	0.53	0.80	0.64	0.92		
h	−0.36	−0.12	0.14	0.17	0.26	0.34	0.62	0.62	0.68		
$t(a)$	−0.27	−1.03	−0.24	−1.93	0.08	0.95	−1.73	−0.34	0.34	1.04	0.405
$t(b)$	64.04	65.82	40.20	63.31	67.96	52.28	45.55	58.48	36.63		
$t(s)$	18.37	19.12	19.86	24.77	21.34	19.47	22.57	25.32	23.47		
$t(h)$	−12.71	−4.90	3.42	5.82	10.61	10.84	15.21	21.64	15.40		
R^2	0.95	0.95	0.89	0.95	0.95	0.92	0.91	0.94	0.88		

after portfolio formation ranges from −0.00 percent for the decile of stocks with the worst short-term past returns (measured from 12 to 2 months before portfolio formation) to 1.31 percent for the decile with the best short-term past

Table VI

Average Monthly Excess Returns (in Percent) on Equal-Weight NYSE Deciles Formed Monthly Based on Continuously Compounded Past Returns

At the beginning of each month t, all NYSE firms on CRSP with returns for months $t - x$ to $t - y$ are allocated to deciles based on their continuously compounded returns between $t - x$ and $t - y$. For example, firms are allocated to the 12-2 portfolios for January 1931 based on their continuously compounded returns for January 1930 through November 1930. Decile 1 contains the NYSE stocks with the lowest continuously compounded past returns. The portfolios are reformed monthly, and equal-weight simple returns in excess of the one-month bill rate are calculated for January 1931 (3101) to December 1993 (9312). The table shows the averages of these excess returns for 6307 to 9312 (366 months) and 3101 to 6306 (390 months).

	Portfolio Formation	Average Excess Returns									
Period	Months	1	2	3	4	5	6	7	8	9	10
6307–9312	12-2	−0.00	0.46	0.61	0.55	0.72	0.68	0.85	0.90	1.08	1.31
6307–9312	24-2	0.36	0.60	0.59	0.66	0.71	0.81	0.73	0.80	0.93	1.05
6307–9312	36-2	0.46	0.60	0.77	0.69	0.73	0.81	0.69	0.78	0.84	0.97
6307–9312	48-2	0.66	0.70	0.77	0.74	0.71	0.71	0.72	0.71	0.72	0.89
6307–9312	60-2	0.86	0.76	0.73	0.75	0.70	0.71	0.74	0.70	0.66	0.73
6307–9312	60-13	1.16	0.81	0.77	0.76	0.74	0.72	0.72	0.73	0.54	0.42
3101–6306	12-2	1.49	1.52	1.32	1.49	1.39	1.45	1.45	1.55	1.58	1.87
3101–6306	24-2	2.24	1.60	1.57	1.70	1.41	1.31	1.32	1.24	1.26	1.46
3101–6306	36-2	2.31	1.74	1.65	1.46	1.40	1.40	1.32	1.23	1.27	1.36
3101–6306	48-2	2.34	1.81	1.62	1.60	1.37	1.30	1.33	1.22	1.24	1.26
3101–6306	60-2	2.49	1.78	1.74	1.50	1.39	1.33	1.27	1.18	1.28	1.14
3101–6306	60-13	2.62	1.85	1.63	1.61	1.43	1.24	1.34	1.28	1.08	1.01

returns. (Skipping the portfolio formation month in ranking stocks reduces bias from bid-ask bounce.)

Table VI also confirms that average returns tend to reverse when portfolios are formed using returns for the four years from 60 to 13 months prior to portfolio formation. For these portfolios, the average return in the month after portfolio formation ranges from 1.16 percent for the decile of stocks with the worst long-term past returns to 0.42 percent for stocks with the best past returns. In the 1963–1993 results, however, long-term return reversal is observed only when the year prior to portfolio formation is skipped in ranking stocks. When the preceding year is included, short-term continuation offsets long-term reversal, and past losers have lower future returns than past winners for portfolios formed with up to four years of past returns.

Can our three-factor model explain the patterns in the future returns for 1963–1993 on portfolios formed on past returns? Table VII shows that the answer is yes for the reversal of long-term returns observed when portfolios are formed using returns from 60 to 13 months prior to portfolio formation. The regressions of the post-formation returns on these portfolios on $R_M - R_f$, SMB, and HML produce intercepts that are close to 0.0 both in absolute terms and on the GRS test. The three-factor model works because long-term past losers

Table VII

Three-Factor Regressions for Monthly Excess Returns (in Percent) on Equal-Weight NYSE Portfolios Formed on Past Returns: 7/63–12/93, 366 Months

$$R_i - R_f = a_i + b_i(R_M - R_f) + s_i\text{SMB} + h_i\text{HML} + e_i$$

The formation of the past-return deciles is described in Table VI. Decile 1 contains the NYSE stocks with the lowest continuously compounded returns during the portfolio-formation period (12-2, 48-2, or 60-13 months before the return month). $t()$ is a regression coefficient divided by its standard error. The regression R^2s are adjusted for degrees of freedom. GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of ten portfolios are all 0.0. p(GRS) is the p-value of GRS.

	1	2	3	4	5	6	7	8	9	10	GRS	p(GRS)
Portfolio formation months are t-12 to t-2												
a	−1.15	−0.39	−0.21	−0.22	−0.04	−0.05	0.12	0.21	0.33	0.59		
b	1.14	1.06	1.04	1.02	1.02	1.02	1.04	1.03	1.10	1.13		
s	1.35	0.77	0.66	0.59	0.53	0.48	0.47	0.45	0.51	0.68		
h	0.54	0.35	0.35	0.33	0.32	0.30	0.29	0.23	0.23	0.04		
$t(a)$	−5.34	−3.05	−2.05	−2.81	−0.54	−0.93	1.94	3.08	3.88	4.56	4.45	0.000
$t(b)$	21.31	33.36	42.03	51.48	61.03	73.62	68.96	62.67	51.75	35.25		
$t(s)$	17.64	16.96	18.59	20.87	22.06	23.96	21.53	19.03	16.89	14.84		
$t(h)$	6.21	6.72	8.74	10.18	11.86	13.16	11.88	8.50	6.68	0.70		
R^2	0.75	0.85	0.89	0.92	0.94	0.96	0.95	0.94	0.92	0.86		
Portfolio formation months are t-48 to t-2												
a	−0.73	−0.32	−0.09	−0.08	−0.05	−0.00	0.07	0.10	0.15	0.37		
b	1.16	1.12	1.06	1.05	1.02	1.01	1.00	0.99	1.04	1.11		
s	1.59	0.87	0.64	0.52	0.48	0.42	0.41	0.40	0.42	0.49		
h	0.90	0.60	0.44	0.44	0.36	0.31	0.18	0.11	−0.05	−0.26		
$t(a)$	−2.91	−2.79	−0.96	−0.99	−0.67	−0.01	1.08	1.46	2.09	3.60	2.02	0.031
$t(b)$	18.61	39.22	46.55	53.19	57.82	63.78	64.72	58.62	57.02	43.37		
$t(s)$	17.91	21.36	19.68	18.61	19.17	18.51	18.52	16.61	16.22	13.40		
$t(h)$	8.91	12.94	11.93	13.78	12.61	11.87	7.34	4.19	−1.55	−6.35		
R^2	0.73	0.88	0.91	0.92	0.93	0.94	0.95	0.93	0.94	0.90		
Portfolio formation months are t-60 to t-13												
a	−0.18	−0.16	−0.13	−0.07	0.00	0.02	0.06	0.10	−0.07	−0.12		
b	1.13	1.09	1.07	1.04	0.99	1.00	1.00	1.01	1.06	1.15		
s	1.50	0.83	0.67	0.59	0.47	0.38	0.35	0.40	0.45	0.50		
h	0.87	0.54	0.50	0.42	0.34	0.29	0.23	0.13	−0.00	−0.26		
$t(a)$	−0.80	−1.64	−1.69	−0.99	0.02	0.40	0.96	1.43	−0.92	−1.36	1.29	0.235
$t(b)$	20.24	44.40	55.03	61.09	63.79	65.68	62.58	58.26	60.49	53.04		
$t(s)$	18.77	23.63	24.09	24.06	21.21	17.44	15.43	16.18	18.06	16.33		
$t(h)$	9.59	13.67	15.94	15.31	13.46	11.82	8.98	4.46	−0.14	−7.50		
R^2	0.75	0.91	0.93	0.94	0.94	0.94	0.94	0.93	0.94	0.93		

load more on SMB and HML. Since they behave more like small distressed stocks, the model predicts that the long-term past losers will have higher average returns. Thus, the reversal of long-term returns, which has produced so much controversy (DeBondt and Thaler (1985, 1987), Chan (1988), Ball and

Kothari (1989), Chopra, Lakonishok, and Ritter (1992)), falls neatly within the predictions of our three-factor model. Moreover, since the model captures the economic essence of long-term winners (strong stocks) and losers (smaller distressed stocks), we speculate that it can explain the stronger reversal of long-term returns observed in the 1931–1963 period (Table VI).

In contrast, Table VII shows that the three-factor model misses the continuation of returns for portfolios formed on short-term past returns. In the three-factor regressions for these portfolios, the intercepts are strongly negative for short-term-losers (low-past-returns) and strongly positive for short-term winners. The problem is that losers load more on SMB and HML (they behave more like small distressed stocks) than winners. Thus, as for the portfolios formed on long-term past returns, the three-factor model predicts reversal for the post-formation returns of short-term losers and winners, and so misses the observed continuation.

As noted earlier, when portfolios are formed on long-term past returns that include the year prior to portfolio formation, short-term continuation offsets long-term reversal, leaving either continuation or little pattern in future returns. Again, however, future returns on long-term losers load more on SMB and HML, so the three-factor model (1) incorrectly predicts return reversal. The regressions in table VII for portfolios formed using returns from two to 48 months prior to portfolio formation are an example.

V. Exploring Three-Factor Models

The tests above suggest that many patterns in average stock returns, so-called anomalies of the CAPM, are captured by the three-factor model of (1). In this section we show that the explanatory returns of the model are not unique. Many other combinations of three portfolios describe returns as well as $R_M - R_f$, SMB, and HML. These results support our conclusion that a three-factor model is a good description of average returns.

We first provide some background. Fama (1994) shows that a generalized portfolio-efficiency concept drives Merton's (1973) ICAPM. Because ICAPM investors are risk averse, they are concerned with the mean and variance of their portfolio return. ICAPM investors are, however, also concerned with hedging more specific state-variable (consumption-investment) risks. As a result, optimal portfolios are multifactor-minimum-variance (MMV): they have the smallest possible return variances, given their expected returns and sensitivities to the state-variables.

In a two-state-variable ICAPM, MMV portfolios are spanned by (they can be generated from) the risk-free security and any three linearly independent MMV portfolios. (With two state variables and a finite number of risky securities, a third MMV portfolio is needed to capture the tradeoff of expected return for return variance that is unrelated to the state variables.) This spanning result has two implications that we test below.

(S1) The expected excess returns on any three MMV portfolios describe the expected excess returns on all securities and portfolios. In other words, the intercepts in regressions of excess returns on the excess returns on any three MMV portfolios are equal to 0.0.

(S2) The realized excess returns on any three MMV portfolios perfectly describe (intercepts equal to 0.0 and R^2 equal to 1.0) the excess returns on other MMV portfolios.

In the usual representation of a three-factor ICAPM, the three explanatory portfolios are the value-weight market and MMV portfolios that mimic the two state variables of special hedging concern to investors. (S1) and (S2) say, however, that any three MMV portfolios can be used to generate MMV portfolios and describe returns.

The tests that follow can also be interpreted in terms of a model in the spirit of Ross' (1976) APT. Suppose (i) investors are risk averse, (ii) there are two common factors in returns, and (iii) the number of risky securities is finite. Fama's (1994) analysis again implies that optimal portfolios are MMV: they have the smallest possible variances given their expected returns and their loadings on the two common factors. With a finite number of securities, however, the returns on MMV portfolios in general are not perfectly explained by the two common factors in returns. As a result, as in the ICAPM, the risk-free security and three MMV portfolios are needed to span MMV portfolios and describe expected returns. Again, (S1) and (S2) hold.

A. *Spanning Tests*

In principle, the explanatory variables in the ICAPM (or the APT) are the expected returns on MMV portfolios in excess of the risk-free rate. SMB and HML in (1) are, however, each the difference between two portfolio returns. Equation (1) is still a legitimate three-factor risk-return relation as long as the two components of SMB (S and B) and the two components of HML (H and L) are MMV. $R_B - R_f$ and $R_L - R_f$ are then exact linear combinations of $R_M - R_f$, $R_S - R_f$, and $R_H - R_f$, so subtracting R_B from R_S (to get SMB) and R_L from R_H (HML) has no effect on the intercepts or the explanatory power of the three-factor regressions.

Obviously, we do not presume that our ad hoc size and book-to-market portfolios are truly MMV. We suggest, however, that if $R_M - R_f$, SMB, and HML do a good job describing average returns, then M, S, B, H, and L are close to MMV. (S1) and (S2) say that this hypothesis has two testable implications. (i) All combinations of three of the portfolios M, S, B, H, and L should provide similar descriptions of average returns (S1). (ii) Realized excess returns on any three of the candidate MMV portfolios should almost perfectly describe the excess returns on other candidate MMV portfolios (S2).

Table VIII tests (S2) with regressions that use the four different triplets of $R_M - R_f$, $R_S - R_f$, $R_H - R_f$, and $R_L - R_f$ to explain the excess return on the excluded MMV proxy. (We drop the big-stock portfolio B from the list of MMV proxies because the correlation between R_M and R_B is 0.99.) The results are consistent with (S2). Excess returns on any three of M, S, H, and L almost

Table VIII

Regressions to Explain Monthly Excess Returns (in Percent) on M, S, L, H, SMB and HML: 7/63–12/93, 366 Months

The portfolios (described in Table I) include the market (M), the small-stock portfolio (S), the low-book-to-market portfolio (L), the high-book-to-market portfolio (H), the difference between H and L (HML), and the difference between S and the return on the big-stock portfolio B (SMB). To simplify the notation, the table uses the portfolio labels, rather than explicit notation for their excess returns. The regression R^2 and the residual standard error, $s(e)$, are adjusted for degrees of freedom. The numbers in parentheses are t-statistics (regression coefficients divided by their standard errors).

							R^2	$s(e)$
S	=	0.28	+1.17 M	+ e			0.79	2.68
		(1.99)	(36.95)					
L	=	−0.10	+1.20 M	+ e			0.92	1.62
		(−1.15)	(62.84)					
H	=	0.46	+0.99 M	+ e			0.80	2.16
		(4.08)	(38.73)					
SMB	=	0.19	+0.21 M	+ e			0.10	2.74
		(1.32)	(6.54)					
HML	=	0.56	−0.21 M	+ e			0.13	2.41
		(4.42)	(−7.53)					
S	=	0.00	−0.83 M	+1.00 L	+0.81 H	+ e	0.99	0.65
		(0.17)	(−29.12)	(46.81)	(50.12)			
L	=	−0.03	+0.86 M	+0.86 S	−0.67 H	+ e	0.99	0.60
		(−0.90)	(51.83)	(46.81)	(−29.30)			
H	=	0.06	+0.98 M	+1.09 S	−1.05 L	+ e	0.98	0.75
		(1.36)	(31.38)	(50.12)	(−29.30)			
M	=	0.00	−0.85 S	+1.03 L	+0.75 H	+ e	0.98	0.66
		(0.08)	(−29.12)	(51.83)	(31.38)			

perfectly describe the excess return on the fourth. The regression intercepts are close to 0.0, and the R^2 values are close to 1.0 (0.98 and 0.99).

Table IX summarizes the intercepts from regressions that use the four different triplets of $R_M - R_f$, $R_S - R_f$, $R_H - R_f$, and $R_L - R_f$ to describe the excess returns on the different sets of portfolios examined in previous sections. As predicted by (S1), different triplets of M, S, L, and H provide equivalent descriptions of returns. Specifically, different three-factor regressions produce much the same GRS tests, mean absolute and squared intercepts, and average values of R^2. Moreover, the regression intercepts (not shown) are nearly identical for different triplets of explanatory returns. Substantively, Table IX says that different three-factor regressions all miss the continuation of returns

Table IX

Summary of Intercepts from One-Factor CAPM Excess-Return Regressions and Different Versions of the Three-Factor ICAPM Regressions: 7/63–12/93, 366 Months

The alternative sets of dependent excess returns (and the tables that describe them) include the 25 size-BE/ME portfolios (Table I), the E/P and five-year sales-rank deciles (Table II), the nine portfolios doubled-sorted on C/P and five-year sales rank (Table IV), the long-term and short-term past return deciles (60-13 and 12-2) (Table VI). The explanatory variables (described in Table I) include the excess returns on the market portfolio (M), the small-stock portfolio (S), the low- and high-book-to-market portfolios (L and H), SMB (the return on S minus the return on the big-stock portfolio B) and HML (H minus L). GRS is the F-statistic of Gibbons, Ross, and Shanken (1989), testing the hypothesis that the regression intercepts for a set of dependent portfolios are all 0.0. p(GRS) is the p-value of GRS. Ave $|a|$ and Ave a^2 are the average absolute and squared values of the intercepts for a set of dependent portfolios, and Ave R^2 is the average of the regression R^2 (adjusted for degrees of freedom).

Dependent Ports.	Explanatory Ports.			GRS	p(GRS)	Ave $\|a\|$	Ave a^2	Ave R^2
25 Size-BE/ME	M			2.76	0.000	0.286	0.1140	0.77
25 Size-BE/ME	M	SMB	HML	1.97	0.004	0.093	0.0164	0.93
25 Size-BE/ME	M	S	H	2.06	0.002	0.097	0.0170	0.93
25 Size-BE/ME	M	S	L	2.16	0.001	0.102	0.0183	0.92
25 Size-BE/ME	M	L	H	1.87	0.008	0.094	0.0159	0.92
25 Size-BE/ME	S	L	H	2.06	0.002	0.094	0.0162	0.92
E/P	M			2.85	0.002	0.260	0.1059	0.83
E/P	M	SMB	HML	0.84	0.592	0.051	0.0039	0.93
E/P	M	S	H	0.95	0.488	0.059	0.0051	0.94
E/P	M	S	L	1.02	0.427	0.064	0.0057	0.94
E/P	M	L	H	0.86	0.575	0.052	0.0041	0.93
E/P	S	L	H	0.86	0.571	0.051	0.0040	0.93
Sales Rank	M			2.51	0.006	0.256	0.0821	0.82
Sales Rank	M	SMB	HML	0.87	0.563	0.053	0.0058	0.93
Sales Rank	M	S	H	1.01	0.437	0.055	0.0068	0.94
Sales Rank	M	S	L	0.96	0.474	0.052	0.0059	0.94
Sales Rank	M	L	H	0.92	0.514	0.052	0.0057	0.93
Sales Rank	S	L	H	0.93	0.509	0.052	0.0057	0.93
C/P & Sales Rank	M			2.93	0.002	0.268	0.1007	0.80
C/P & Sales Rank	M	SMB	HML	1.04	0.405	0.062	0.0068	0.93
C/P & Sales Rank	M	S	H	1.13	0.338	0.067	0.0068	0.93
C/P & Sales Rank	M	S	L	1.14	0.333	0.063	0.0064	0.93
C/P & Sales Rank	M	L	H	1.03	0.416	0.061	0.0064	0.92
C/P & Sales Rank	S	L	H	1.05	0.396	0.061	0.0065	0.93
60-13	M			2.51	0.006	0.268	0.0899	0.80
60-13	M	SMB	HML	1.29	0.235	0.092	0.0114	0.92
60-13	M	S	H	1.38	0.186	0.094	0.0112	0.92
60-13	M	S	L	1.19	0.299	0.077	0.0074	0.92
60-13	M	L	H	1.29	0.234	0.089	0.0102	0.91
60-13	S	L	H	1.30	0.230	0.090	0.0107	0.91
12-2	M			5.13	0.000	0.337	0.1647	0.79
12-2	M	SMB	HML	4.46	0.000	0.331	0.2097	0.90
12-2	M	S	H	4.45	0.000	0.322	0.2027	0.90
12-2	M	S	L	4.58	0.000	0.329	0.2040	0.90
12-2	M	L	H	4.51	0.000	0.326	0.2047	0.90
12-2	S	L	H	4.46	0.000	0.328	0.2069	0.90

for portfolios formed on short-term past returns. On the other hand, every triplet of M, S, L, and H does a similar and excellent job describing the returns on the LSV deciles formed on E/P and sales rank, and the LSV portfolios double-sorted on C/P and sales rank. In results not shown in Table IX, excellent three-factor descriptions of returns are also obtained for the LSV BE/ME and C/P deciles, and for portfolios double-sorted on sales rank and BE/ME or E/P. Finally, Table IX shows that all triplets of M, S, L, and H capture the reversal of returns for portfolios formed on long-term past returns.

Table IX says that our original (FF 1993) combination of the market, SMB, and HML fares no better or worse than triplets of M, S, H, and L. But the original set of portfolios has one advantage. Table X shows that $R_M - R_f$, SMB, and HML are much less correlated with one another than $R_M - R_f$, $R_S - R_f$, $R_B - R_f$, $R_H - R_f$, and $R_L - R_f$. This makes three-factor regression slopes easier to interpret, and it is why we use $R_M - R_f$, SMB, and HML in the regressions of Tables I, III, V, and VII.

B. Additional MMV Proxies

M, S, H, and L are not the only portfolios that give equivalent descriptions of returns. We construct explanatory portfolios (MMV proxies) that are simple averages of the returns for the bottom and top three deciles of each of the LSV (BE/ME, E/P, C/P, and sales-rank) sorts and the short- and long-term past-return sorts. For example, the high E/P return (HE/P) is the average of the top three E/P decile returns.

The MMV proxies formed from the LSV BE/ME, E/P, and C/P deciles work much like our L and H (low- and high-BE/ME) portfolios in describing returns. The reason is clear from Table X. Excess returns on the LSV low BE/ME, E/P, and C/P portfolios are correlated 0.99 with each other, and they are correlated 0.98 with our L (low-BE/ME) portfolio. Excess returns on the LSV high BE/ME, E/P, and C/P portfolios are correlated 0.98 and 0.99 with each other, and their correlations with our H portfolio are 0.97 and 0.98. The "high" portfolios are much more correlated with one another than with the "low" portfolios. The MMV proxies produced by the LSV BE/ME, E/P, and C/P sorts also have similar average excess returns, 0.48 to 0.51 for the three "low" portfolios and 0.97 to 1.03 for the three "high" portfolios. These returns are a bit higher than those of our L and H portfolios, 0.44 and 0.90, probably because L and H are constructed from value-weight components.

In short, the "low" and "high" MMV proxies from the LSV BE/ME, E/P, and C/P sorts mimic our L and H portfolios. Thus it is not surprising that they can replace L and H in the three-factor model. Without showing the details, combining the market portfolio M with LBE/ME and HBE/ME, or LE/P and HE/P, or LC/P and HC/P produces three-factor descriptions of returns like those in Table IX.

Ball (1978) argues that scaling stock prices with accounting variables, like earnings, cash flow, or book equity, is a good way to extract the information in stock prices about expected returns. Our tests suggest, more precisely, that

Table X

Average Monthly Excess Returns (in Percent) and Correlations of Excess Returns for MMV Proxies: 7/63–12/93, 366 Months

The market portfolio (M), the small-stock portfolio (S), the low- and high-book-to-market portfolios (L and H), SMB (the return on S minus the return on the big-stock portfolio B) and HML (H minus L) are described in Table I. LBE/ME, LE/P, LC/P, and LSR are the simple averages of the returns on the three lowest LSV BE/ME, E/P, C/P, and five-year-sales-rank deciles, while HBE/ME, HE/P, HC/P, and HSR are the simple averages of the returns on the three highest BE/ME, E/P, C/P, and five-year-sales-rank deciles, described in Table II. L60-13 and H60-13 are the simple averages of the returns on the three lowest and highest long-term-past-return deciles, described in Table VI.

	L	LBE/ME	LE/P	LC/P	HSR	H60-13	H	HBE/ME	HE/P	HC/P	LSR	L60-13	M	SMB	HML
						Average Excess Returns									
Means	0.44	0.48	0.51	0.49	0.60	0.56	0.90	1.03	0.97	1.00	0.90	0.91	0.45	0.28	0.46
Std. Dev.	5.56	5.55	5.63	5.58	5.77	5.47	4.87	5.66	5.18	5.29	5.40	6.41	4.43	2.89	2.59
t (Mn)	1.51	1.67	1.74	1.69	2.00	1.95	3.55	3.47	3.58	3.60	3.18	2.72	1.93	1.88	3.42
						Correlations									
LBE/ME	0.98														
LE/P	0.98	0.99													
LC/P	0.98	0.99	0.99												
HSR	0.97	0.98	0.98	0.98											
H60-13	0.97	0.98	0.97	0.98	0.97										
H	0.88	0.89	0.91	0.90	0.94	0.91									
HBE/ME	0.86	0.87	0.90	0.89	0.93	0.88	0.97								
HE/P	0.88	0.90	0.90	0.91	0.95	0.92	0.97	0.98							
HC/P	0.88	0.89	0.90	0.90	0.95	0.90	0.98	0.99	0.99						
LSR	0.89	0.91	0.93	0.93	0.94	0.91	0.95	0.97	0.95	0.96					
L60-13	0.85	0.86	0.89	0.88	0.91	0.84	0.93	0.97	0.93	0.95	0.97				
M	0.96	0.96	0.95	0.95	0.94	0.95	0.90	0.84	0.88	0.87	0.87	0.81			
SMB	0.53	0.51	0.55	0.54	0.57	0.52	0.56	0.66	0.60	0.61	0.63	0.67	0.32		
HML	−0.48	−0.44	−0.40	−0.41	−0.31	−0.36	−0.02	−0.02	−0.07	−0.04	−0.13	−0.06	−0.37	−0.10	

MMV proxies formed on E/P, C/P, and BE/ME mimic more or less the same combinations of the underlying common factors in returns.

Unlike the proxies created from the LSV BE/ME, E/P, and C/P sorts, MMV proxies constructed from the LSV sales-rank sort, or from long-term past returns, cannot successfully replace L and H in tests of the three-factor model. There are two possible explanations. (i) Perhaps sorts on sales growth or long-term past return expose variation in expected returns missed by sorts on size, BE/ME, E/P, and C/P. The fact that the three-factor regressions in Table IX have no problem explaining the average returns on the sales-rank and long-term-past-return deciles seems to refute this hypothesis. (ii) The sales-rank and long-term-past-return proxies are not diversified enough. If the proxies are not close to MMV, too much of their return variance is not priced. This diversifiable risk creates an errors-in-variables problem that contaminates tests of three-factor models.

C. The CAPM versus Three-Factor Models

Table IX shows tests of the CAPM in which $R_M - R_f$ is used alone to explain returns. The GRS test always rejects the CAPM at the 0.99 level (p-values less than 0.01). Omitting the details, which are similar to FF (1992) and LSV (1994), the CAPM fails because univariate market βs show little relation to variables like BE/ME, E/P, C/P, and sales rank, that are strongly related to average return. Table IX also shows that, except for portfolios formed on short-term past return, where all models fail, the CAPM is dominated by the three-factor model. The average absolute pricing errors (intercepts) of the CAPM are large (25 to 30 basis points per month), and they are three to five times those of the three-factor model (5 to 10 basis points per month).

Using the ICAPM to interpret the problems of the CAPM is instructive. Fama (1994) shows that the multifactor-minimum-variance (MMV) portfolios that are relevant for ICAPM investors can be characterized as combinations of Markowitz' (1959) mean-variance-efficient (MVE) portfolios and MMV mimicking portfolios for the state variables. Most important, a market equilibrium in the ICAPM implies that the market portfolio M (the aggregate of the MMV portfolios chosen by investors) is MMV. But M almost surely is not MVE. Thus, market βs do not suffice to explain expected returns. More specifically, because ICAPM investors have different tastes for state-variable risks and general sources of return variance, the market βs of some or all MMV state-variable mimicking portfolios cannot explain their expected returns. This means that β alone cannot explain the expected returns on all MMV portfolios.

In contrast, in the CAPM all sources of return variance, including the state-variable or common-factor risks of the ICAPM and the APT, are equivalent to investors. Investors hold mean-variance-efficient portfolios, and the market portfolio is MVE. This means that the expected excess returns on all securities and portfolios, including MMV portfolios, are fully explained by their market βs. Thus, one way to test whether a multifactor return process collapses to CAPM rather than multifactor ICAPM or APT pricing is to test whether the expected excess returns on MMV portfolios are explained by their market βs.

Table VIII shows CAPM time-series regressions in which $R_M - R_f$ is used alone to explain the excess returns on our MMV proxies S, L, and H. The MMV proxies that are seriously mispriced by the CAPM are prime candidates for explaining why three-factor models improve on the CAPM's description of average returns. Table VIII says that the CAPM misprices the low-book-to-market portfolio L by -0.10 percent per month ($t = -1.15$). The pricing error for the small-stock portfolio S is more serious, 0.28 percent per month ($t = 1.99$). The largest CAPM pricing error is for the high-book-to-market portfolio H. The one-factor CAPM regression intercept for H is 0.46 percent per month ($t = 4.08$). The CAPM regressions for SMB and HML confirm that H's high return is the prime embarrassment of the CAPM. Much of the discussion of competing interpretations of our results that follows focuses on stories for H's (or HML's) average return.

VI. Interpreting the Results

Standard tests of the CAPM ask whether loadings on a market proxy can describe the average returns on other portfolios. Algebraically, these are just tests of whether the market proxy is in the set of mean-variance-efficient (MVE) portfolios that can be formed from the returns to be explained (Fama (1976), Roll (1977), Gibbons, Ross, and Shanken (1989)). Similarly, tests of a three-factor ICAPM or APT ask whether loadings on three portfolios can describe the average returns on other portfolios. Such tests in effect ask whether the explanatory portfolios span the three-factor MMV portfolios that can be formed from the returns to be explained (Fama (1994)). Thus, a minimalist (purely algebraic) interpretation of our results is that the portfolios M, S, B, H, and L are in the sets of three-factor-MMV portfolios that can be formed from sorts on size, BE/ME, E/P, C/P, sales rank, and long-term past returns. But our explanatory portfolios cannot span the three-factor-MMV portfolios that can be constructed from sorts on short-term past returns.

The economic interpretation of our results is more contentious. We distinguish three stories. The first says that asset pricing is rational and conforms to a three-factor ICAPM or APT that does not reduce to the CAPM (FF (1993, 1994, 1995)). The second story agrees that a three-factor model describes returns, but argues that it is investor irrationality that prevents the three-factor model from collapsing to the CAPM. Specifically, irrational pricing causes the high premium for relative distress (the average HML return). Proponents of this view include Lakonishok, Shleifer, and Vishny (1994), Haugen (1995), and MacKinlay (1995). The third story says the CAPM holds but is spuriously rejected because (i) there is survivor bias in the returns used to test the model (Kothari, Shanken, and Sloan (1995)), (ii) CAPM anomalies are the result of data snooping (Black (1993), MacKinlay (1995)), or (iii) the tests use poor proxies for the market portfolio.

A. The Case for a Multifactor ICAPM or APT

In FF (1992) we reject the CAPM based on evidence that size and book-to-market-equity (BE/ME) capture cross-sectional variation in average returns that is missed by univariate market βs. We have since tried to infer whether these size and book-to-market effects are generated by a multifactor ICAPM or APT.

One necessary condition for multifactor ICAPM or APT pricing is multiple common (undiversifiable) sources of variance in returns. FF (1993) show that there is indeed covariation in returns related to size and BE/ME (captured by loadings on SMB and HML), above and beyond the covariation explained by the market return. Moreover, FF (1995) show that there are common factors in fundamentals like earnings and sales that look a lot like the SMB and HML factors in returns.

The acid test of the three-factor model is whether it can explain differences in average returns. FF (1993) find that the model describes the average returns on portfolios formed on size and BE/ME. It may not be surprising, however, that portfolios like SMB and HML that are formed on size and BE/ME can explain the returns on other portfolios formed on size and BE/ME (albeit with a finer grid). We address this concern here by testing whether the three-factor model can explain other prominent CAPM average-return anomalies. We find that the patterns in average return produced by forming portfolios on E/P, C/P, sales growth, and long-term past return are absorbed by the three-factor model, largely because they line up with the loadings of the portfolios on HML. The tests of (1) on industries in FF (1994) are also a check on FF (1993).

The three-factor model (1) is also useful in applications. For example, Reinganum (1990) finds that size-adjusted average returns are higher for NYSE stocks than for NASD stocks. Fama, French, Booth, and Sinquefield (1993) use (1) to explain this puzzling result. Controlling for size, NYSE stocks have higher loadings on HML, and thus higher predicted returns. Carhart (1994) finds that the three-factor model (1) provides sharper evaluations of the performance of mutual funds than the CAPM. SMB adds a lot to the description of the returns on small-stock funds, and loadings on HML are important for describing the returns on growth-stock funds. FF (1994) find that the three-factor model (1) signals higher costs of equity for distressed industries than for strong industries, largely because the distressed industries have higher loadings on HML.

One can argue that all of this still falls within a minimalist interpretation of the three-factor model; that is, we have simply found three portfolios that provide a parsimonious description of returns and average returns, and so can absorb most of the anomalies of the CAPM. In other words, without knowing why, we have stumbled on explanatory portfolios that are close to three-factor MMV. And the main reason many will not go beyond this minimalist story is clear. We have not identified the two state variables of special hedging concern to investors that lead to three-factor asset pricing. Such state variables are necessary in a three-factor ICAPM or APT, if they are not to collapse to the CAPM.

FF (1993) interpret the average HML return as a premium for a state-variable risk related to relative distress. This story is suggested by the evidence in FF (1995) that low book-to-market-equity is typical of firms that have persistently strong earnings, while high-BE/ME is associated with persistently low earnings. Moreover, FF (1994) argue that the variation through time in the loadings of industries on HML correctly reflects periods of industry strength or distress. Industries have strong positive HML loadings in bad times and negative loadings when times are good. Finally, Chan and Chen (1991) present evidence for a risk factor in returns and average returns related to relative-distress.

Why is relative distress a state variable of special hedging concern to investors? One possible explanation is linked to human capital, an important asset for most investors. Consider an investor with specialized human capital tied to a growth firm (or industry or technology). A negative shock to the firm's prospects probably does not reduce the value of the investor's human capital; it may just mean that employment in the firm will expand less rapidly. In contrast, a negative shock to a distressed firm more likely implies a negative shock to the value of specialized human capital since employment in the firm is more likely to contract. Thus, workers with specialized human capital in distressed firms have an incentive to avoid holding their firms' stocks. If variation in distress is correlated across firms, workers in distressed firms have an incentive to avoid the stocks of all distressed firms. The result can be a state-variable risk premium in the expected returns of distressed stocks.

Unfortunately, tracing a common factor in returns to an economic state variable does not in itself imply that the state variable is of special hedging concern to investors, and so carries a special risk premium. For example, in Mayers (1972), covariation with the income return on (nonmarketable) human capital has no special premium. Jagannathan and Wang (1995) argue that human capital (taken to be marketable) is just another asset in the CAPM. Thus, even if we found two state variables that could explain the common variation in returns tracked by portfolios like SMB and HML, we would still face the problem of explaining why the state variables produce special premiums. Merton (1973) clearly recognizes this problem. It lurks on the horizon in all tests of multifactor ICAPM's or APT's.

B. *The Distress Premium Is Irrational*

Lakonishok, Shleifer, and Vishny (LSV 1994), Haugen (1995), and MacKinlay (1995) argue that the premium for relative distress, the difference between the average returns on high- and low-book-to-market stocks, is too large to be explained by rational pricing. Indeed, LSV and Haugen conclude that the premium is almost always positive and so is close to an arbitrage opportunity. Table XI, which shows the annual R_M-R_f, SMB, and HML returns for 1964–1993, provides relevant evidence.

Table XI

Annual Three-Factor Explanatory Returns: $R_M - R_f$, SMB, and HML, 1964–1993, $N = 30$

R_M is the annual market return. R_f is the return obtained by rolling over 12 one-month bills during a year. SMB is the difference between the annual returns on the small-stock portfolio, S, and the big-stock portfolio, B. HML is the difference between the annual returns on the high-book-to-market portfolio, H, and the low-book-to-market portfolio, L. The portfolios M, S, B, H, and L are defined in Table I. t(Mean) is the mean of the annual returns (Mean) divided by its standard error (Std. Dev.)/$29^{1/2}$. Negative is the number of negative annual returns.

Year	R_M-R_f	SMB	HML
1964	13.25	1.15	6.32
1965	10.31	22.84	12.54
1966	−13.87	2.47	3.12
1967	22.01	50.84	−6.69
1968	7.92	23.89	16.97
1969	−16.12	−14.14	−8.86
1970	−5.35	−10.98	23.35
1971	11.46	6.46	−12.54
1972	13.92	−12.40	3.39
1973	−22.40	−23.13	19.35
1974	−34.93	0.17	11.18
1975	31.72	16.85	7.34
1976	21.61	13.19	26.01
1977	−7.91	22.32	8.58
1978	2.33	13.97	−0.05
1979	14.52	19.18	−3.21
1980	23.23	6.31	−23.86
1981	−16.91	7.03	24.32
1982	11.78	8.58	12.76
1983	14.66	15.31	20.00
1984	−4.58	−7.90	18.64
1985	24.06	0.17	0.12
1986	9.98	−8.11	8.46
1987	−1.51	−11.99	−1.03
1988	14.31	5.46	14.76
1989	23.74	−12.86	−5.92
1990	−11.28	−15.02	−11.07
1991	28.49	14.34	−14.20
1992	5.73	6.39	22.71
1993	8.07	7.20	17.44
Mean	5.94	4.92	6.33
Std. Dev.	16.33	15.44	13.11
t(Mean)	1.96	1.72	2.60
Negative	10	9	10

If the premium for relative distress is close to an arbitrage opportunity, the standard deviation of HML should be small. In fact, HML's standard deviation, 13.11 percent per year, is similar to the standard deviations of $R_M - R_f$ and SMB, 16.33 percent and 15.44 percent per year, respectively. The average values of the three annual premiums are also similar: 6.33 percent for HML, 5.94 percent for $R_M - R_f$, and 4.93 percent for SMB. The yearly returns confirm that a high-book-to-market strategy is not a sure thing. HML is negative in ten of the thirty years we study, $R_M - R_f$ is also negative ten times, and SMB is negative nine times. In short, if the relative-distress premium is too high to be explained by rational asset pricing, one must also be suspicious of the market and size premiums.

But the fact that the premium for relative distress is not an arbitrage opportunity does not imply that it is rational. LSV and Haugen argue that the premium is due to investor over-reaction. Specifically, investors do not understand that the low earnings growth of high-BE/ME firms and the high earnings growth of low-BE/ME firms quickly revert to normal levels after portfolios are formed on BE/ME. FF (1995) argue, however, that over-reaction cannot be the whole story, since the high distress premium in returns persists for at least five years after portfolio formation, but the mean reversion of earnings growth is apparent much sooner.

Another LSV argument is that the relative-distress premium is irrational because periods of poor returns on distressed stocks are not typically periods of low GNP growth or low overall market returns. Since the relative-distress premium is not related to these obvious macroeconomic state variables, they conclude that the premium arises simply because investors dislike distressed stocks and so cause them to be underpriced.

The essence of a multifactor model, however, is that covariance with the market return is not sufficient to measure risk. Moreover, our industry work leans us toward the conclusion that the state variable related to relative distress is not a common macro-variable, like GNP. FF (1994) find that industries fluctuate between strength and distress. The expansions and contractions of the economy are minor compared to the variation in the fortunes of industries. We suspect that product innovation, technology shocks, and changes in tastes dramatically alter the relative prospects of industries without having much effect on aggregate variables like GNP. We also suspect that industries provide a muted version of the changes in the relative prospects of individual firms. (The evidence of Davis and Haltiwanger (1992) that variation in aggregate employment is trivial relative to the gross job creation and destruction that occurs, in good times and bad, at the level of individual firms is consistent with this view.) In other words, although two unidentified state variables lead to common risk factors in returns, they are not the market factor and we should not expect to find their tracks in variables that are important in generating the market factor. Thus, we are not surprised by the LSV evidence that variation in a return spread like HML is not highly correlated with GNP, or with the market return itself.

Finally, LSV argue that the relative distress premium is irrational because diversified portfolios of high- and low-book-to-market firms have similar return variances. Equation (1) provides an explanation. The positive HML slopes of high-BE/ME (distressed) firms raise their return variances and imply higher average returns. The negative HML slopes of low-BE/ME (strong) firms also raise their return variances but imply lower average returns. In any case, in a multifactor ICAPM or APT, different sources of return variance do not carry the same premiums, so variance is not a sufficient statistic for a portfolio's risk.

C. The Distress Premium Is Spurious

The final category of stories for the high relative-distress premium in average returns says that the CAPM holds and the premium is the spurious result of (i) survivor bias, (ii) data snooping, or (iii) a bad proxy for the market portfolio in tests of the CAPM.

Survivor Bias—Kothari, Shanken, and Sloan (KSS 1995) are the prime proponents of a survivor-bias story. They argue that average returns on high-book-to-market portfolios of COMPUSTAT stocks like H are overstated because COMPUSTAT is more likely to include distressed firms that survive and to miss distressed firms that fail. The direct evidence of Chan, Jegadeesh, and Lakonishok (1995) refutes this claim. Moreover, KSS concede that survivor bias is not a major problem for value-weight portfolios, which means that it cannot explain why the high average return of H (or HML) is not captured by the CAPM.

Data Snooping—Lo and MacKinlay (1988), Black (1993), and MacKinlay (1995) argue that CAPM anomalies may be the result of data-snooping. A nontrivial portion of asset pricing research is devoted to dredging for anomalies. As the profession rummages through the same data, we are sure to find patterns in average returns, like the size and book-to-market effects, that are inconsistent with the CAPM, but are sample specific. In this view, it is not surprising that factors like SMB and HML, that are aimed directly at the spurious anomalies, produce multifactor models that "explain" the anomalies in the same data used to unearth them. The data-snooping story predicts that in out-of-sample tests, average SMB and HML (more specifically, average S and H) returns will fall to levels that are consistent with their market βs. Our three-factor model will then reduce to a CAPM in which, like the expected returns on all other securities and portfolios, the expected returns on the MMV mimicking portfolios for the three common factors will be completely explained by their market βs.

Data-snooping bias can never be ruled out, but we suggest four counter arguments. (i) Davis (1994) shows that the distress premium is not special to the post-1962 COMPUSTAT period studied in FF (1992, 1993). Using a sample of large firms, he finds a strong relation between BE/ME and average return from 1941 to 1962. (ii) Tests on international data, which can also be regarded as out-of-sample, produce relations between average return and variables like size, BE/ME, E/P, and C/P much like those observed in U.S. data (e.g., Chan, Hamao, and Lakonishok (1991), Capaul, Rowley, and Sharpe (1993)). (iii) Ball (1978) argues that scaled versions of price like size, BE/ME, E/P, and C/P are proxies for expected return. They are thus excellent for identifying the real failures of asset pricing models like the CAPM. (iv) Our results suggest that data-snooping has not been that effective; there are not so many independent average-return anomalies to explain. Specifically, the message from our results is that, whatever the economic explanation, a three-factor model captures the CAPM anomalies produced by sorts on size, BE/ME, E/P, C/P, sales rank, and long-term past return.

Bad Market Proxies—Finally, there is the ritual argument that the CAPM holds, and its average-return anomalies just expose the shortcomings of empirical proxies for the market portfolio. In this view, multifactor models are just a convenient way to recover CAPM expected returns. Specifically, the spanning result (S1) implies that the loadings on any X linearly independent X-factor-MMV portfolios produce the same expected returns for securities and portfolios as their univariate βs on a mean-variance-efficient portfolio. Thus, if the CAPM holds and the unobserved market portfolio is MVE, any X linearly independent X-factor-MMV portfolios can be used in a multifactor model to recover CAPM expected returns.

Unfortunately, the bad-market-proxy argument does not justify the way the CAPM is currently applied, for example, to estimate the cost of capital or to evaluate portfolio managers. The bad market proxies that produce spurious anomalies in tests of the CAPM are similar to those used in applications. If the common market proxies are not MVE, applications that use them rely on the same flawed estimates of expected return that undermine tests of the CAPM. In the end, the irony of the bad-market-proxy argument is that if the CAPM is true but the market portfolio is unobservable, multifactor models like ours may provide better estimates of CAPM expected returns.

D. The Continuation of Short-Term Returns

We have saved until last the discussion of the main embarassment of the three-factor model, its failure to capture the continuation of short-term returns documented by Jegadeesh and Titman (1993) and Asness (1994). There are at least three possible stories.

(i) This particular anomaly is a spurious result of data snooping. The weak continuation of short-term returns in the 1931–1963 period preceding our asset pricing regressions is suggestive (Table 6). Jegadeesh and Titman (1993) show, however, that weak continuation is limited to the 1930's. They find short-term return continuation in the 1941–1964 and post-1964 periods. Still, the fact that the continuation of short-term returns is so far from the contrarian spirit of other CAPM anomalies (like the size, BE/ME, E/P, C/P, and sales-growth effects, or the reversal of long-term returns) suggests that further out-of-sample tests, for example on international data, are desirable.

(ii) Asset pricing is irrational. Investors underreact to short-term past information, which produces return continuation, but they overreact to long-term past information, which produces return reversal (Lakonishok, Shleifer, and Vishny (1994), Haugen (1995)). Behavioral-finance types should be wary of this explanation. The evidence of Kahneman and Tversky (1982) and others, which forms the foundation of existing behavioral finance models, predicts overreaction and return reversal. (See, for example, DeBondt and Thaler

(1985).) The continuation of short-term returns is then as much a challenge to behavioral finance as to our asset-pricing model.

(iii) Asset pricing is rational, but our three-factor model is (alas!) just a model, and the continuation anomaly exposes one of its shortcomings. In this view, future work should look for a richer model, perhaps including an additional risk factor, that encompasses the continuation of short-term returns. We are reluctant to follow this track, however, until robustness checks of the continuation anomaly have run their course.

VII. Conclusions

Fama and French (1993) find that the three-factor risk-return relation (1) is a good model for the returns on portfolios formed on size and book-to-market-equity. We find that (1) also explains the strong patterns in returns observed when portfolios are formed on earnings/price, cash flow/price, and sales growth, variables recommended by Lakonishok, Shleifer, and Vishny (1994) and others. The three-factor risk-return relation (1) also captures the reversal of long-term returns documented by DeBondt and Thaler (1985). Thus, portfolios formed on E/P, C/P, sales growth, and long-term past returns do not uncover dimensions of risk and expected return beyond those required to explain the returns on portfolios formed on size and BE/ME. Fama and French (1994) extend this conclusion to industries.

The three-factor risk-return relation (1) is, however, just a model. It surely does not explain expected returns on all securities and portfolios. We find that (1) cannot explain the continuation of short-term returns documented by Jegadeesh and Titman (1993) and Asness (1994).

Finally, there is an important hole in our work. Our tests to date do not cleanly identify the two consumption-investment state variables of special hedging concern to investors that would provide a neat interpretation of our results in terms of Merton's (1973) ICAPM or Ross' (1976) APT. The results of Chan and Chen (1991) and Fama and French (1994, 1995) suggest that one of the state variables is related to relative distress. But this issue is far from closed, and multiple competing interpretations of our results remain viable.

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